KIX 1001: ENGINEERING MATHEMATICS 1
Tutorial 10: Multiple Integrals

1. Evaluate the following:

   a)
   $$\int_1^2 \int_2^3 (2x - y)dy\,dx$$
   Ans: 0.5

   b)
   $$\int_1^5 \int_{-1}^2 (x - 5y)dx\,dy$$
   Ans: -174

   c)
   $$\int_1^3 \int_2^5 (2x - 3y)\,dx\,dy$$
   Ans: 6

2. Evaluate each of the following integrals over the given region D:

   a)
   $$\iint_D \left(1 - \frac{1}{2}x^2 - \frac{1}{2}y^2\right)dA \quad \text{where D is the region given by } 0 \le x \le 1, 0 \le y \le 1$$

   Ans: 2/3

   b)
   $$\iint_D (4xy - y^3)dA \qquad \text{where D is the region given by } y = x^{0.5} \text{ and } y = x^3$$

   Ans: 55/156

3. Evaluate the following integral:

   $$\iint_D 42y^2 - 12x\,dA \qquad where\ D = \{(x, y)|0 \le x \le 4, (x - 2)^2 \le y \le 6\}$$

   Ans: 11136

4. Evaluate the following integral over the indicated rectangle (a) by integrating with respect to x first and (b) with respect to y first.

   $$\iint_R 12x - 18y\,dA \qquad\qquad R = [-1, 4] \times [2, 3]$$

   Ans for (a) and (b): -135

5. Compute the following double integral over the indicated rectangle

a) $$\iint_R 6y\sqrt{x} - 2y^3 \, dA \qquad\qquad R = [1, 4] \times [0, 3]$$

Ans: 9/2

b) $$\iint_R ye^{y^2 - 4x} \, dA \qquad\qquad R = [0, 2] \times [0, \sqrt{8}]$$

Ans: 372.37

6. Evaluate:

a) $$\int_{-3}^{1} \int_{-1}^{2} \int_{0}^{1} 6 \, xyz \, dy \, dx \, dz$$

Ans: -18

b) $$\int_{0}^{4} \int_{-2}^{-1} \int_{1}^{2} xy \, dx \, dy \, dz$$

Ans: -9

7. Evaluate:

a) $$\iiint_B xyz^2 \, dV \qquad\qquad \begin{array}{c} 0 \le x \le 1 \\ -1 \le y \le 2 \\ 0 \le z \le 3 \end{array}$$

Ans: 27/4

b) $$\iiint_B 4x^2y - z^3 \, dz \, dy \, dx \qquad\qquad \begin{array}{c} 2 \le x \le 3 \\ -1 \le y \le 4 \\ 0 \le z \le 1 \end{array}$$

Ans: $-755/4$

8. Integrate the function $(x, y, z) = xy$ over the volume enclose by the planes $z = x + y$ and $z = 0$, and between the surfaces $y = x^2$ and $x = y^2$.

Ans: 3/28

9. Find the mass for the following square lamina, represented by the unit square (shown below) with variable density $\rho(x, y) = (x + y + 2)$ g/cm^2.
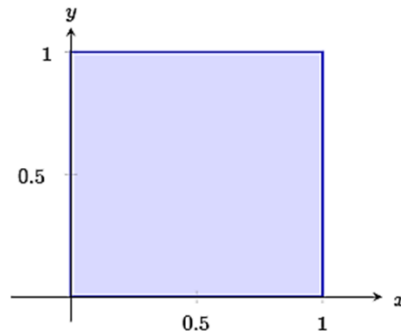

Figure 1. A region R representing a lamina

Ans: 3 g

10. Find the mass and center of mass for the following solid region Q bounded by $x + 2y + 3z = 6$ and the coordinate planes (as shown below) and has density $\rho(x, y, z) = x^2yz$.
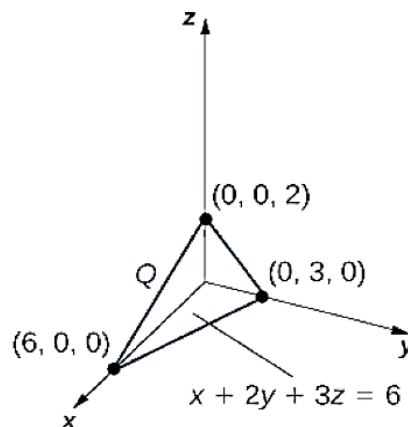

Figure 2: Finding the mass of a three-dimensional solid Q.

Ans: Mass = 108/35 g and the center of mass = (2.25, 0.75, 0.5)