KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 11: Multiple Integrals in Polar Coordinate & Its Engineering Application

1. In the following exercises, change the cartesian integral into an equivalent polar coordinate integral. Then solve the integral in polar coordinate:

   a) $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$

   b) $\int_{0}^{2}\int_{0}^{x} y\, dydx$

   c) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\; dydx$

   d) $\int_{0}^{ln\,2}\int_{0}^{\sqrt{(ln2)^2-y^2}} e\sqrt{x^2+y^2}dxdy$

2. Evaluate the $\iint 1-x^2-y^2\; dA$ using polar coordinates

3. Find the volume below $z=\frac{y^2}{x^2+y^2}$, above xy-plane and between cylinder $x^2+y^2=1$

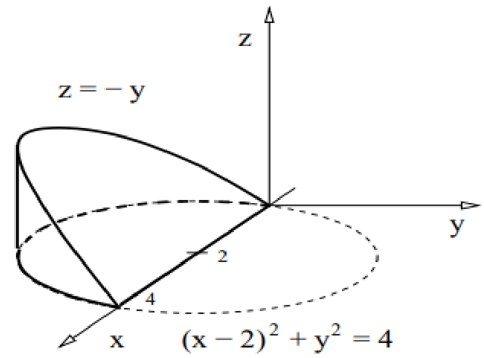
and $x^2+y^2=2$

4. Find the volume between the sphere $x^2+y^2+z^2=1$ and the cone $z=\sqrt{x^2+y^2}$

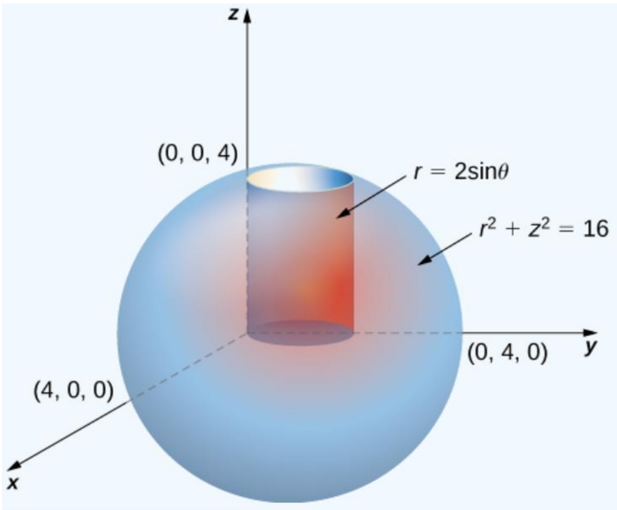
5. Volume is equal to area only if the height (z) is equal to 1. Find the area of 'R' where 'R' is the region bound by r=3 cosθ.

6. $\iiint y\;\; dV$, a solid is bound by $z=4-x^2-y^2$ in the first octant ($x=0, y=0, z=0$).

7. Use cylindrical coordinates to find the volume of a curved wedge cut out from a cylinder $(x^2-2)^2+\;y^2=4$ by the planes $z=0$ z = 0 and $z=\;-y$.

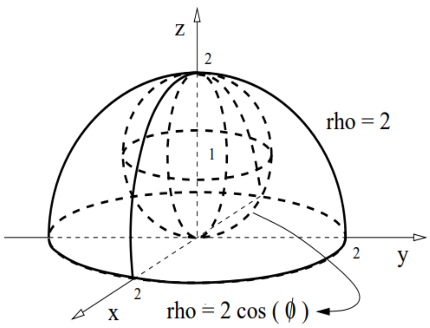


8. Consider the region E inside the right circular cylinder with equation r=2sinθ, bounded below by the rθ -plane and bounded above by the sphere with radius 4 centered at the origin. Set up a triple integral over this region with a function f(r,θ,z) in cylindrical coordinates.



9. Find the volume of solid bound by $z = 2\ and\ z = \sqrt{x^2 + y^2}$

10. Use spherical coordinates to find the volume of the region outside the sphere ρ = 2 cos(ϕ) and inside the sphere ρ = 2 with ϕ ∈ [0, π/2].



11. Given a solid bound by $z = 2$ and $z = \sqrt{x^2 + y^2}$, find the mass density if the mass density is directly proportional to the square of the distance from origin.

12. Find the mass of 'T', $\rho\ (x, y, z) = y$, where T is region bound by $y = x^2 + \ z^2$ and $y = 4$.