KIX1001: ENGINEERING MATHEMATICS 1
TUTORIAL 12: LINE INTEGRALS

1) If $A = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C A \bullet dr$ from (0, 0, 0) to (1, 1, 1) along the following paths C:

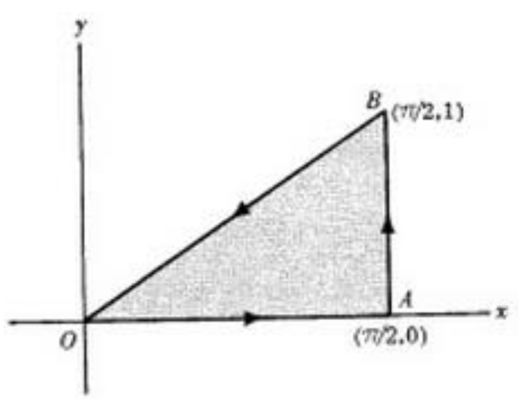
(a) $x = t, y = t^2, z = t^3$.
(b) The straight line from (0, 0, 0) to (1, 0, 0) then to (1, 1, 0), and then to (1, 1, 1).
(c) The straight line joining (0, 0, 0) and (1, 1, 1).
[Ans: (a) 5; (b) 23/3; (c) 13/3]

2) Find the work done in moving a particle in a force field given by $F = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.
[Ans: 303]

3) Determine the work done in moving a particle counterclockwise once around a circle C in the *xy*-plane, if the circle has center at the origin and radius of 3 and if those field is given by $F = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$
[Ans: 18π]

4) Fine the line integral for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ in the positive direction in traversing C. $y = x$ and $y = x^2$ intersect at (0, 0) and (1, 1).
[Ans: -1/20]

5) Evaluate $\oint_C (y - \sin x)dx + \cos x dy$ where C is the triangle of the figure below.

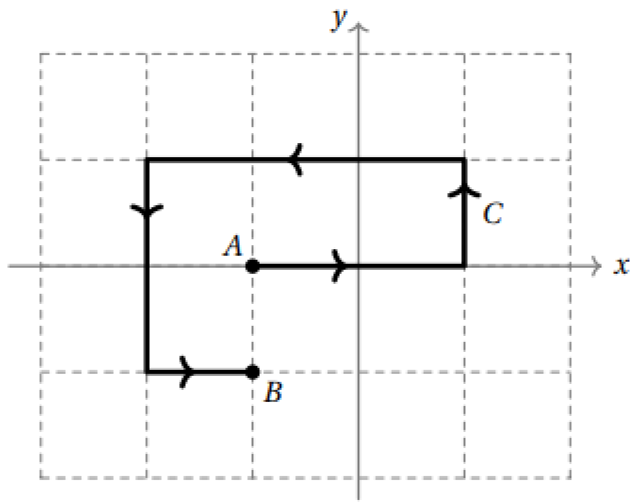


[Ans: $-\frac{2}{\pi} - \frac{\pi}{4}$]

6) Calculate $\oint_C -x^2 y\,dx + xy^2\,dy$ where C is the circle of radius 2 centered on the origin.

[Ans: 8π]

7) Compute the line integral of $\boldsymbol{F}(x, y) = \langle x^3, 4x\rangle$ along the path C shown below against a grid of unit-sized squares.



[Ans: 20]

8) A particle starts at (-2,0) and moves along the x-axis to (2,0). Then it moves along the upper part of the circle $x^2 + y^2 = 4$ and back to (-2,0). Compute the work done on this particle by the force field $\boldsymbol{F}(x, y) = \langle x, x^3 + 3xy^2\rangle$.

[Ans: 12π]

9) Let $\boldsymbol{F}(x, y) = \langle \sin x, \cos y\rangle$ and let C be the curve that is the top half of the circle $x^2 + y^2 = 1$, traversed counterclockwise from (1,0) to (-1,0), and the line segment from (-1,0) to (-2,3). Evaluate the line integral $\int \boldsymbol{F}.\boldsymbol{T}ds = \int_C \sin x\ dx + \cos y\ dy$.

[Ans: cos(1)-cos(2)+sin(3)]