# KIX1001: ENGINEERING MATHEMATICS 1

## TUTORIAL 13: SURFACE INTEGRALS

1. Evaluate the surface integral of the vector field $\mathbf{F} = 3x^2\mathbf{i} - 2yx\mathbf{j} + 8\mathbf{k}$ over the surface *S* that is the graph of $z = 2x - y$ over the rectangle [0,2]x[0,2].
   [Ans: -8]

2. Let *S* be the triangle with vertices (1,0,0), (0,2,0) and (0,1,1) and let $\mathbf{F} = xyz(\mathbf{i} + \mathbf{j})$. calculate the surface integral
   $$\iint_S \mathbf{F}.d\mathbf{S}$$
   If the triangle is oriented by the "downward" normal.
   [Ans: -1/10]

3. The equations $z = 12$, $x^2 + y^2 \leq 25$ describe a disk of radius 5 lying in the plane z=12. Suppose that is the position vector field $\mathbf{r}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Compute $\iint_S \mathbf{r}.d\mathbf{S}$ .
   [Ans: $300\pi$]

4. Let S be the closed surface that consists of the hemisphere $x^2 + y^2 + z^2 = 1 \geq 0$ , and its base $x^2 + y^2 \leq 1$, z=0. Let *E* be the electric field defined by $\mathbf{E}(x,y,z) = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$ . Find the electric flux across S.
   [Ans: $4\pi$]

5. Find the area of the ellipse cut on the plane $2x + 3y + 6z = 60$ by the circular cylinder $x^2 + y^2 = 2x$.
   [Ans: $\frac{7\pi}{6}$]

6. Find the integral $\iint_S \ x \ dS$, where the surface $S$ is the part of the sphere $x^2 + y^2 + z^2 = a^2$ lying in the first octant.
   [Ans: $\frac{\pi a^3}{4}$]

7. Find the integral $\iint_S \ \frac{dS}{\sqrt{x^2+y^2+z^2}}$ , where $S$ is the part of the cylindrical surface parameterized by $r(u,v) = (a\ \cos u\ , a \sin u\ , v)$, $0 \leq u \leq 2\pi, 0 \leq v \leq H$.
   [Ans: $2\pi a\ \ln \frac{H+\sqrt{a^2+H^2}}{a}$]