KIX1001: ENGINEERING MATHEMATICS 1

TUTORIAL14: STOKES' THEOREM

1. Suppose $\mathbf{F} = \langle -y, x, z \rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 25$ below the plane z=4, oriented with the outward-pointing normal (so that the normal at (5,0,0) is **i**). Compute the flux integral $\iint_S curl\, \mathbf{F}.d\mathbf{S}$ using Stokes' theorem
   [Ans: -18π]

2. Use Stokes' theorem to evaluate $\iint_S curl\, \mathbf{F}.d\mathbf{S}$ where $F = z^2\mathbf{i} - 3xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the part of $z = 5 - x^2 - y^2$ above the *z=1*. Assume that *S* is oriented upwards.
   [Ans: 0]

3. Use Stokes' theorem to evaluate $\int_C \mathbf{F}.d\mathbf{r}$ where $\mathbf{F} = z^2\mathbf{i} + y^2\mathbf{j} + x\mathbf{k}$ and *C* is the triangle with vertices (1,0,0), (0,1,0) and (0,0,1) with counter clockwise rotation.

   [Ans: -1/6]

4. Verify Stokes' Theorem for the field $\boldsymbol{F} = \langle x^2, 2x, z^2 \rangle$ on the ellipse $S = \{(x, y, z): 4x^2 + y^2 \leq 4, z = 0\}$
   [Ans: 4π]

5. Verify Stokes' Theorem for $\boldsymbol{F} = \langle y^2, -x, 5z \rangle$ and S is the paraboloid $z = x^2 + y^2$ with the circle $x^2 + y^2 = 1$ as its boundary.
   [Ans: $-\pi$]

6. Use Stokes' Theorem to calculate $\iint (\nabla \times \boldsymbol{F}) \cdot \hat{n} dS$ for $\boldsymbol{F} = \langle xz^2, x^3, \cos(xz) \rangle$ where S is the part of the ellipsoid $x\ x^2 + y^2 + 3z^2 = 1$ below the xy-plane and $\hat{n}$ is the lower normal.
   [Ans: $-\frac{3\pi}{4}$]

7. Use Stoke's Theorem to evaluate the line integral $\int \boldsymbol{F} \cdot d\boldsymbol{r}\ _c$ where $\boldsymbol{F}$ is the vector $\boldsymbol{F} = \left(4e^{x^2} - y\right)\boldsymbol{i} + (16\sin(y^2) + 3x)\boldsymbol{j} + (4y - 2x - e^z)\boldsymbol{k}$ and C is the curve of intersection of the cylinder $x^2 + y^2 = 16$ and the plane $z = 2x + 4y$ and C is oriented in a counterclockwise direction when viewed from above.
   [Ans: 320π]

8. Evaluate the line integral of $\boldsymbol{F}(x, y, z) = \langle xy, 2z, 3y \rangle$ over the curve C that is the intersection of the cylinder $x^2 + y^2 = 9$ with the plane $x + z = 5$.
[Ans: 9π]

9. Evaluate $\iint (\nabla \times \boldsymbol{F}) \cdot \boldsymbol{n} dS$ where $\boldsymbol{F}(x, y, z) = \langle yz, xz, xy \rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.
[Ans: 0]