# KIX 1001: ENGINEERING MATHEMATICS 1

## Tutorial 2: Partial Derivatives & Engineering Applications of Partial Derivatives

1. Find the partial derivative ($\frac{\partial f}{\partial y}$) and ($\frac{\partial f}{\partial x}$) of these functions using the limit definition
   (a) $f(x,y) = x^2y + 2x + y^3$ [Ans: $f_x = 2xy + 2$, $f_y = x^2 + 3y^2$]
   (b) $f(x,y) = x^2 - 4xy + y^2$ [Ans: $f_x = 2x - 4y$, $f_y = -4x + 2y$]
   (c) $f(x,y) = 2x^3 + 3xy - y^2$ [Ans: $f_x = 6x^2 + 3y$, $f_y = 3x - 2y$]

2. Determine all the first and second order partial derivatives of the function
   (a) $f(x,y) = x^2y^3 + 3y + x$ [Ans: $\frac{\partial^2 f}{\partial x \partial y} = 6xy^2$]
   (b) $f(x,y) = x^4 \sin 3y$ [Ans: $\frac{\partial^2 f}{\partial x \partial y} = 12x^3 cos 3y$]
   (c) $f(x,y) = x^2y + \ln(y^2 - x)$ [Ans: $\frac{\partial^2 f}{\partial x \partial y} = 2x + \frac{2y}{(y^2-x)^2}$
   (d) $f(x,y) = e^{xy}(2x - y)$ [Ans: $\frac{\partial^2 f}{\partial x \partial y} = e^{xy}[2x^2y - xy^2 + 4x - 2y]$

3. Find both partial derivatives for each of the following two variables functions
   (a) $g(x,y) = ye^{x+y}$
   (b) $h(x,y) = x \sin y - y \cos x$
   (c) $p(x,y) = x^y + y^2$
   (d) $U(x,y) = \frac{9y^3}{x-y}$

4. For f(x,y,z), use the implicit function theorem to find dy/dx and dy/dz
   (a) $f(x,y,z) = x^2y^3 + z^2 + xyz$
   (b) $f(x,y,z) = x^3z^2 + y^3 + 4xyz$
   (c) $f(x,y,z) = 3x^2y^3 + xz^2y^2 + y^3zx^4 + y^2z$

5. Find $\frac{\partial F}{\partial s}$ and $\frac{\partial F}{\partial t}$ ,if applicable, for the following composite functions
   (a) $F = \sin(x + y)$ where $x = 2st$ and $y = s^2 + t^2$ [Ans: $\frac{\partial F}{\partial s} = 2\cos((s+t)^2)(s+t)$]
   (b) $F = \ln(x^2 + y)$ where $x = e^{(s+t2)}$ and $y = s^2 + t$ [Ans: $\frac{\partial F}{\partial s} = \frac{1}{e^{2s+2t^2}+s^2+t}(e^{2s+2t^2} + s)$]
   (c) $F = x^2y^2$ where $x = s \cos t$ and $y = s \sin t$ [Ans: $\frac{\partial F}{\partial s} = 4s^3(\frac{1}{2} sin 2t)^2$]
   (d) $F = xy + yz^2$ where $x = e^t$, $y = e^t \sin t$ and $z = e^t \cos t$ [Ans: $\frac{\partial F}{\partial t} = e^{2t}(2sint + cost) + e^{3t}(cos^3 t + 3sintcos^2 t - 2sin^2 tcost)$]

6. Find dy/dx and dy/dz (if applicable) for each of the following
   (a) $7x^2 + 2xy^2 + 9y^4 = 0$
   (b) $x^3z^2 + y^3 + 4xyz = 0$
   (c) $3x^2y^3 + xz^2y^2 + y^3zx^4 + y^2z = 0$
   (d) $y^5 + x^2y^3 = 1 + y \exp(x^2)$

7. (a) In polar coordinates, x = r cos θ, y = r sin θ, show that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$
(b) Obtain the Jacobian J of the transformation s = 2x + y, t = x – 2y and determine the inverse of the transformation $J_1$. Confirm that $J_1=J^{-1}$.
(c) Show that if x + y = u and y = uv, then $\frac{\partial(x,y)}{\partial(u,v)} = u.$
(d) Verify whether the functions $u = \frac{x+y}{1-xy}$ and v = $\tan^{-1}x + \tan^{-1}y$ are functionally dependent.
(e) If x = uv, $= \frac{u+v}{u-v}$ , find $\frac{\partial(u,v)}{\partial(x,y)}$. [Ans: $\frac{(u-v)^2}{4uv}$]

8.
(a) How sensitive is the volume V = $\pi r^2 h$ of a right circular cylinder to small changes in its radius and height near the point $(r_0, h_0)$ = (1,3)?
(b) If r is measured with an accuracy of ±1% and h with an accuracy of ±0.5%, about how accurately can V be calculated from the formula V = $\pi r^2 h$? [Ans: 0.025 or 2.5%]
(c) The period T of a simple pendulum is T = $2\pi\sqrt{\frac{l}{g}}$, find the maximum percentage error in T due to possible errors up to 1% in *l* and 2% in g (Hint: $\frac{dl}{l}$ = 0.001 and $\frac{dg}{g}$ = 0.002) [Ans: 1.5%]
(d) The range R of a projectile which starts with a velocity v at an elevation α is given by $R = \frac{v^2 sin2\alpha}{g}$. Find the percentage error in R due to an error of 1% in v and an error of 0.5% in α. [Ans: 2 + α cot 2α]

9. Find the equations of the tangent plane and normal line to the following surfaces at the points indicated:
(a) $x^2 + 2y^2 + 3z^2 = 6$ at (1,1,1) [Ans: x + 2y + 3z = 6 (eqn of tangent plane)]
(b) $2x^2 + y^2 - z^2 = -3$ at (1,2,3) [Ans: 2x + 2y – 3z = -3 (eqn of tangent plane)]
(c) $x^2 + y^2 - z = 1$ at (1,2,4) [Ans: 2x +4y – z = 6 (eqn of tangent plane)]
(d) $\ln\left(\frac{x}{y}\right) - z^2(x - 2y) - 3z = 3$ at (4,2,-1) [Ans: -3/4 x + 3/2 y – 3z = 3]
(e) $x^3z + z^3x - 2yz = 0$ at (1,1,1) [Ans: 2x – y + z = 2]
(f) $z = 5+(x-1)^2 + (y+2)^2$ at (2,0,10) [Ans: 2x + 4y – z = -6]
(g) $\frac{x^2}{12} + \frac{y^2}{6} + \frac{z^2}{4} = 1$ at (1,2,1) [Ans: x + 4y + 3z = 2]
(h) $ze^x+e^{z+1}+xy+y=3$ at (0,3-1) [Ans: 2x + y + 2z = 1]