KIX 1001: ENGINEERING MATHEMATICS 1
Tutorial 3: Vector Algebra I

1. Sketch the two points $P(3, -1, 5)$ and $Q(2, 1, -1)$ in three-dimensional space and find the distance between the two points. (Ans: $\sqrt{41}$)

2. For the position vector $\underset{\sim}{a} = \langle 2, 4 \rangle$, compute $3\underset{\sim}{a}, \frac{1}{2}\underset{\sim}{a}$, and $-2a$. Sketch all four vectors on the same axis system. Discuss the effects of scalar multiplication on the magnitude and direction of the original vector.

3. Two vectors are given as $\overrightarrow{OP} = \underset{\sim}{i} + 3\underset{\sim}{j} - 7\underset{\sim}{k}$ and $\overrightarrow{OQ} = 5\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$

   i. Find the unit vector in the direction of $\overrightarrow{PQ}$ (Ans: $\frac{1}{9\sqrt{2}}(4, -5, 11)$)

   ii. Find the direction cosines of $\overrightarrow{PQ}$

   iii. Find the vector with magnitude of 5 in the direction of $\overrightarrow{QP}$ in polar form (Ans: $5\langle \cos(108.32°), \cos(66.87°), \cos(149.8°) \rangle$)

4. Two points are given as $A(1,2)$ and $B(3,4)$.

   i. Find the vector equation of line L that is passing through point A and B. (Ans: $\langle 1,2 \rangle + t\langle 2,2 \rangle$)

   ii. Sketch the line for $t = 0:1:5$ and indicate its direction and initial point.

5. If a unit vector $\vec{a}$ makes an angle of $\pi/3$ with i, $\pi/4$ with j and acute angle θ with k, find θ and the components of $\vec{a}$. (Ans: $60°$ or $120°$, $\frac{1}{2}i + \frac{1}{\sqrt{2}}j + \frac{1}{2}k$ or $\frac{1}{2}i + \frac{1}{\sqrt{2}}j - \frac{1}{2}k$)

6. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, find $|\vec{x}|$. (Ans: 3)

7. Find the gradient for $f = (2x^2 + y)/(x^2 - y^2)$. (Ans: $\frac{2xy(-2y-1)}{(x^2-y^2)^2}i + \frac{x^2+4x^2y+y^2}{(x^2-y^2)^2}j$)

8. Calculate the divergence of the following vector fields of $F(x, y)$ and $G(x, y)$;

   (a) $F = y^3 i + xy j$ (Ans: x)

   (b) $G = \frac{4y}{x^2}i + \sin(y)j + 3k$ (Ans: $-\frac{8y}{x^3} + \cos(y)$)

   (c) $G = e^x i + \ln(xy)j + e^{xyz}k$ (Ans: $e^x + \frac{1}{y} + xye^{xyz}$

9. Calculate the curl of the following vector fields of $F(x, y, z)$;

   (a) $F = 3x^2 i + 2z j - x k$ (Ans: $-2i + j$)

   (b) $F = y^3 i + xy j - z k$ (Ans: $(y - 3y^2)k$)

   (c) $F = (1 + y + z^2)i + (e^{xyz})j - (xyz)k$
      (Ans: $(xz - xye^{xyz})i + (2z - yz)j + (yze^{xyz} - 1)k$)