KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 4: Vector Algebra II

1. Two long straight pipes are specified using Cartesian coordinates as follow:

Pipe A: diameter 0.8; axis through points (2,5,3) and (7,10,8).
Pipe B: diameter 1.0; axis through points (0,6,3) and (-12,0,9).

Explain if the two pipes require realignment to prevent intersection.

2. Determine if the sets of vectors given are parallel or non-parallel. Show your answers:

(i) $\underset{\sim}{a} = \langle 2,4,-1\rangle$ and $\underset{\sim}{b} = \langle -6,-12,3\rangle$ (Ans: Parallel)

(ii) $\underset{\sim}{a} = \langle 4,10\rangle$ and $\underset{\sim}{b} = \langle 2,-9\rangle$ (Ans: Not parallel)

3. Find the unit vectors that are perpendicular to the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ as following:

(i) $\underset{\sim}{a} = \langle 2,4,5\rangle$, $\underset{\sim}{b} = \langle 1,2,-2\rangle$ (Ans : $\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0\rangle$)

(ii) $\underset{\sim}{a} = \langle 2,4,-4\rangle$, $\underset{\sim}{b} = \langle 1,2,-2\rangle$ (Ans: Show that it doesn't exist)

4. Find the distance from $P = (-3,7,4)$ to the line *l* with vector equation; (Ans: $\frac{3\sqrt{323}}{5}$)

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}.$$

5. Calculate the distance between the lines *l* and *m* having vector equations $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{u}$ and $\boldsymbol{r} = \boldsymbol{b} + \mu \boldsymbol{v}$ respectively, where; (Ans: $\frac{3\sqrt{6}}{2}$)

$$\boldsymbol{a} = \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}, \boldsymbol{u} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \text{ and } \boldsymbol{v} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}.$$

6. Find the following equation of line for the line L passing through the point $P(3,1,-2)$ and $Q(-2,7,-4)$.

(i) Vector equation (Ans: $r = \langle 3,1,-2\rangle + t\langle -5,6,-2\rangle$
(ii) Parametric equation (Ans: $x = 3-5t, y = 1+6t, z = -2-2t$)
(iii)Cartesian equation (Ans: $\frac{x-3}{-5} = \frac{y-1}{6} = \frac{z+2}{-2}$)

7. Find the Cartesian equation of plane contains the point (1,2,-1) and perpendicular to the intersecting line of the planes and $2x + y + z = 2$ and $x + 2y + z = 3$. (Ans: $-x - y + 3z = -6$)

8. Find the Cartesian equation of plane contains the line L_1: $\boldsymbol{r_1} = \boldsymbol{a} + t\boldsymbol{u} = \langle 1,-3,4\rangle + \langle 2,1,1\rangle t$ and parallel to the line L_2: $\boldsymbol{r_2} = \boldsymbol{b} + s\boldsymbol{v} = \langle 0,0,0\rangle + \langle 1,2,3\rangle s$. Then, proof that the plane is parallel to line L_2? (Ans: $x - 5y + 3z = 28$)

9. Find the Cartesian equation of plane contains the line L_1: $\boldsymbol{r_1} = \boldsymbol{a} + t\boldsymbol{u} = \langle -2,3,4\rangle + \langle 1,2,-1\rangle t$ and line L_2: $\boldsymbol{r_2} = \boldsymbol{b} + s\boldsymbol{v} = \langle 3,4,0\rangle + \langle -1,-2,1\rangle s$. (Ans: $7x + y + 9z = 25$)

10. Let $\underset{\sim}{a} = \langle 1,-2,-3\rangle$, $\underset{\sim}{b} = \langle 2,1,-1\rangle$ and $\underset{\sim}{c} = \langle 1,3,-2\rangle$. Find

(i) $\underset{\sim}{a} \cdot \underset{\sim}{b}(\underset{\sim}{a} \times \underset{\sim}{b})$ (Ans : $39\boldsymbol{i} - 3\boldsymbol{j} + 15\boldsymbol{k}$)

(ii) $(\underset{\sim}{a} + \underset{\sim}{b}) \times \underset{\sim}{c}$ (Ans : $\langle 27,1,5\rangle$)

11. Determine the shortest distance between the 3D skew lines where; (Ans: $\frac{11}{\sqrt{11}}$)

$$L_a: r_a = 4\boldsymbol{i} + 2\boldsymbol{j} - 6\boldsymbol{k} + t(2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$$

$$L_b: r_b = \boldsymbol{i} - 3\boldsymbol{j} - 3\boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k})$$