KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 6: Matrix Algebra for Non-Homogeneous Linear Algebraic System

1. Solve the unknown $\boldsymbol{F}$ for the following equation:

$$\frac{1}{4}\left[\frac{220}{trace(\boldsymbol{C})}\mathbf{A}^{\boldsymbol{T}}.\mathbf{B}-\mathbf{C}\right]=\frac{1}{3}\boldsymbol{EF}-\frac{20}{determinant(\mathbf{D})}\mathbf{D}$$

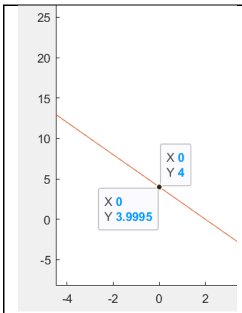
$where\ \mathbf{A}=\begin{bmatrix}1 & 0 & 2\\ 3 & -1 & -3\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}4 & 6 & 8\\ -2 & 5 & 7\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}10 & 11 & 3\\ -10 & -27 & -25\\ 70 & -29 & -27\end{bmatrix}$

$\mathbf{D}=\begin{bmatrix}1 & 2 & 0\\ -1 & 3 & 0\\ 0 & 0 & 4\end{bmatrix},\ \mathbf{E}=\begin{bmatrix}2 & -2 & 1\\ 1 & 2 & 2\\ 2 & 1 & -2\end{bmatrix}$

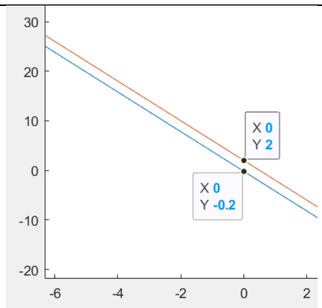
$\frac{1}{3}\boldsymbol{E}=orthogonal\ matrix\ if\ \left(\frac{1}{3}\boldsymbol{E}\right)^{-1}=\left(\frac{1}{3}\boldsymbol{E}\right)^{T}$, verify if $\frac{1}{3}\boldsymbol{E}$ is an orthogonal matrix or not, hence use it for the calculation.

2. Choose the correct graph for each case, then comment on the condition of the coefficient matrix in terms of its determinant and the characteristic of the solution without solving it.

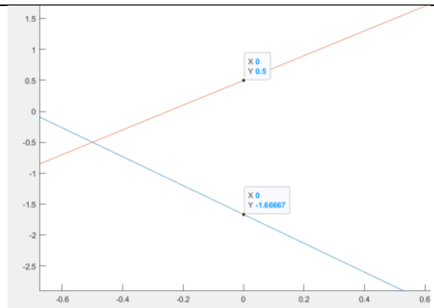
Case 1	Case 2	Case 3
$3x_2+7x_1=-5$ $x_2-2x_1=\frac{1}{2}$	$x_2+2x_1=4$ $2x_2+3.999x_1=7.999$	$5x_2+20x_1=-1$ $x_2+4x_1=2$



(a)



(b)



(c)

3. Continue to solve Q2 by using Cramer's rule. Hence, verify the solution.

4. $[A]\{x\} = \{b\}$ where $[A]$ = coefficient matrix that represents the physical system, while $\{x\}$ and $\{b\}$ are the unknown and known parameters respectively. Compute the solution with the parameters measured with noise using Cramer's rule. Comment on the solution if it is accurately computed & discuss why.

Case	Actual parameters with no noise	Measured parameters with noise
1	$\begin{bmatrix} 3 & 7 \\ 1 & -2 \end{bmatrix}\{x\} = \begin{Bmatrix} -5 \\ 0.5 \end{Bmatrix},\ \{x\} = \begin{Bmatrix} -0.5 \\ -0.5 \end{Bmatrix}$	$\begin{bmatrix} 3 & 7 \\ 1 & -2 \end{bmatrix}\{x\} = \begin{Bmatrix} -5.001 \\ 0.4999 \end{Bmatrix}$
2	$\begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix}\{x\} = \begin{Bmatrix} 4 \\ 7.999 \end{Bmatrix},\ \{x\} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$	$\begin{bmatrix} 3 & 7 \\ 1 & -2 \end{bmatrix}\{x\} = \begin{Bmatrix} 4.001 \\ 7.998 \end{Bmatrix}$

5. Perform GEwPP to solve the following linear algebraic equations. The calculation must involves the scaling, partial pivoting, forward elimination and the backward substitution procedures.

$$y + z + 2t = 0$$
$$x + 2y + 3z + t = 5$$
$$x + 2y + z = 4$$
$$2x + y + z + t = 3$$

6. Obtain the following information for the following matrices.
   (i) Row echelon form (REF).
   (ii) Reduced row echelon form (RREF).
   (iii) Number of linear independent vector.
   (iv) Rank & check if it is full rank or rank deficient matrix.

Matrix **A**	Matrix **B**
$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{6}{7} \end{bmatrix}$	$\begin{bmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ 2 & 6 & 9 \end{bmatrix}$

7. An electronics company produces transistors, resistors and computer chips. Each transistors requires 4 units of copper, 1 unit of zinc, and 2 units of glass. Each resistor requires 3, 3, and 1 unit(s) of the three materials, respectively. Each computer chip requires 2, 1, and 3 unit(s) of the three materials, respectively. The amounts of materials available are 960 units of copper, 510 units of zinc, and 610 units of glass in week 1. The amounts of materials available are 960 units of copper, 510 units of zinc, and 610 units of glass in a week. By using GEwPP, calculate the number of transistors, resistors and computer chips produced per day in average by considering 5 working days per week.

8. A civil engineer involved in construction requires 4800, 5800, and 5700 m^3 of sand, fine gravel and coarse gravel, respectively, for a building project. There are three pits from which these materials can be obtained. The composition of these pits is given below. How much cubic meters must be hauled from each pit in order to meet the engineer's needs? Use Cramer's Rule to solve it.

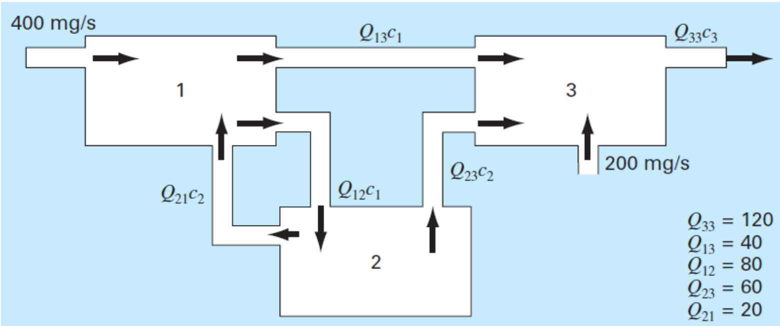
	Sand %	Fine Gravel %	Coarse Gravel %
Pit 1	52	30	18
Pit 2	20	50	30
Pit 3	25	20	55

9. Figure below shows three reactors linked by pipes. The mass balance equations for the reactors are shown below:

For steady state flow: Mass flow rate input = Mass flow rate output

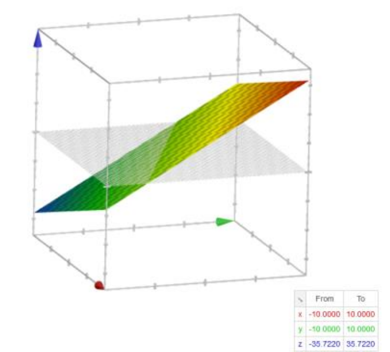
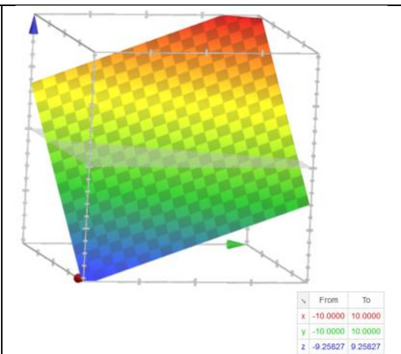
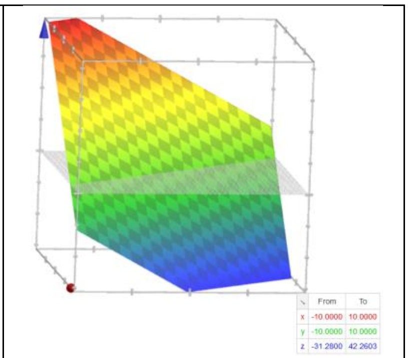
For example, 400 mg/s + $Q_{21}c_2 = Q_{13}c_1 + Q_{12}c_1$ for reactor 1

,where Q is flow rate in m^3/s , c is concentration of the reactor mg/m^3



Continue to develop the linear algebraic equations for the reactors 2 and 3. Then, solve the concentrations of the reactors by using the Naïve GE.

10. Given $z = a_0 + a_1x + a_2y$ is a plane equation, where a is the constant coefficient. 3 planes are given below, please suggest a suitable method based on the determinant analysis and continue to find the intersection point/ line of the planes, if it exists.

 $z = \frac{-6-3x-6y}{-3}$	 $z = \frac{2-4x+2y}{6}$	 $z = -2 - 3x - 2y$

Q1: Solution: $\boldsymbol{F}\begin{bmatrix} -23 & -16/3 & -25/3 \\ -13 & 97/3 & 121/3 \\ 23 & -17/3 & -41/3 \end{bmatrix}$

Q2: Case 1 – (c); Case 2 – (a); Case 3 – (b)

Hint:

Distinct slope or determinant $\neq 0$ indicates a well-conditioned matrix, it has unique solution

Close slope or determinant ≈ 0 indicates ill-conditioned matrix, it is sensitive to noise and might have many solutions.

Equal slope or determinant =0 indicates singular matrix, it has no solution if no interception or infinite solutions if there are infinite interception points.

Q3: Case 1- $\begin{Bmatrix} -0.5 \\ -0.5 \end{Bmatrix}$; Case 2- $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$; Case 3- no solution

Hint: Verified if LHS=RHS

Q4: $\{x\}_{noise} = \begin{Bmatrix} -0.5002 \\ -0.5001 \end{Bmatrix}$; $\{x\}_{noise} = \begin{Bmatrix} -3.999 \\ 4.000 \end{Bmatrix}$

Hint: The effect of noise depending on the condition of the system, e.g. well-conditioned/ ill-conditioned. Compare $\{x\}_{noise}$ & $\{x\}_{true}$

Q5:

$$\begin{Bmatrix} x \\ y \\ z \\ t \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{Bmatrix}$$

Q6: Matrix **A** = Full Rank Matrix**;** Matrix **B** = Rank Deficient Matrix

Q7: $\begin{Bmatrix} 24 \\ 20 \\ 18 \end{Bmatrix}$

Q8: $\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 4005.8 \\ 7131.4 \\ 5162.8 \end{Bmatrix}$

Q9:

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{Bmatrix} 4 \\ 4 \\ 5 \end{Bmatrix} mg/m^3$$

Q10:

$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} -t \\ -1+t \\ t \end{Bmatrix}$, *where* $-\infty \leq t \leq \infty$

It is an intersecting 3D line