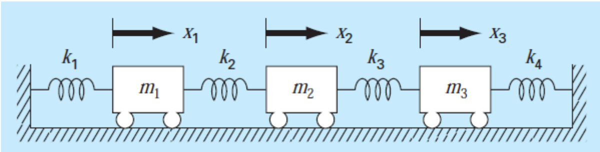
KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 7: Matrix Algebra for Homogeneous Linear Algebraic System

1. The following linear algebraic equations represents the 3 dof mass spring systems below:



$$\begin{bmatrix} \frac{k_1+k_2}{m_1} & -\frac{k_2}{m_1} & 0 \\ -\frac{k_2}{m_2} & \frac{k_2+k_3}{m_2} & -\frac{k_3}{m_2} \\ 0 & -\frac{k_3}{m_2} & \frac{k_3+k_4}{m_3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} - \omega^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Determine the smallest eigenvalue and the corresponding eigenvector for the eigenvalue/ eigenvector problem if $k_1 = k_4 = 15\frac{N}{m}, k_2 = k_3 = 35\frac{N}{m}$, and $m_1 = m_2 = m_3 = 1.5kg$. Then, draw the eigenvector.

2. Continue from Q1, determine the second largest eigenvalue and the corresponding normalised eigenvector. Then, draw the eigenvector.

3. Continue from Q1, determine the largest eigenvalue and the corresponding unscaled eigenvector. Then, draw the eigenvector.

4. Combining the results obtained from Q1-Q4, obtain the eigenvector matrix, P and diagonalize the following matrix. Comment the relationship between the diagonal matrix and the eigenvalue.

$$\begin{bmatrix} \frac{k_1+k_2}{m_1} & -\frac{k_2}{m_1} & 0 \\ -\frac{k_2}{m_2} & \frac{k_2+k_3}{m_2} & -\frac{k_3}{m_2} \\ 0 & -\frac{k_3}{m_2} & \frac{k_3+k_4}{m_3} \end{bmatrix}$$

5. Determine $\begin{bmatrix} \frac{k_1+k_2}{m_1} & -\frac{k_2}{m_1} & 0 \\ -\frac{k_2}{m_2} & \frac{k_2+k_3}{m_2} & -\frac{k_3}{m_2} \\ 0 & -\frac{k_3}{m_2} & \frac{k_3+k_4}{m_3} \end{bmatrix}^{50}$ and comment on the change of it's eigenvalue and eigenvector, as compared to Q4.

6. Given $\mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and $|\mathbf{B}| = 2$ has an eigenvalue of 2. Find the remaining eigenvalues and develop the characteristic equation without developing the eigenvalue problem and without performing the determinant.

7. Continue Q6, using Cayley-Hamilton theorem to verify that:

   $\mathbf{B}^{-1} = \frac{1}{2}\mathbf{B}^2 + (-2)\mathbf{B} + \left(\frac{5}{2}\right)\mathbf{I}$ and $\mathbf{B}^6 = \left[(57)\mathbf{B}^2 + (-108)\mathbf{B} + 52\mathbf{I}\right]$. Then, compute the $\mathbf{B}^5$ via the theorem.

8. Explain why the following equation : $\mathbf{B}^5 = \mathbf{PD^5P}^{-1}$ is not working to solve the Q7 problem?

9. Based on Q7 and Q8, discuss the advantage and disadvantage of diagonalization formula versus the Cayley-Hamilton theorem in solving the power of a matrix.

10. Find all the eigenvalue and normalized eigenvectors in terms of eigenvalue matrix and eigenvector matrix for the matrix $\mathbf{C}$.

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

   Then, verify the eigenvalue matrix and eigenvector matrix if they satisfy the eigenvalue/eigenvector problem, i.e. $(\mathbf{C} - \lambda\mathbf{I})\boldsymbol{x} = \mathbf{0}$.

Q1: Solution: *Eigenvector,* $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_{\lambda=6.3350} = \begin{Bmatrix} 1 \\ 1.1571 \\ 1 \end{Bmatrix}$

Q2: Solution: *Normalised Eigenvector,* $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_{\lambda=33.3333\text{ ,normalised}} = \frac{1}{\sqrt{2}}\begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} -0.7071 \\ 0 \\ 0.7071 \end{Bmatrix}$

Q3 Solution: *Eigenvector,* $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_{\lambda=73.6650} = \begin{Bmatrix} 1 \\ -1.7285 \\ 1 \end{Bmatrix}$

Q4 Solution: $\mathbf{D} = \begin{bmatrix} 6.335 & 0 & 0 \\ 0 & 33.3333 & 0 \\ 0 & 0 & 73.665 \end{bmatrix}$

Q5 Solution:

$$\mathbf{A}^{50} = 10^{93}\begin{bmatrix} 0.46255450 & -0.79952578 & 0.46255450 \\ -0.79952578 & 1.38198087 & -0.79952578 \\ 0.46255450 & -0.79952578 & 0.46255450 \end{bmatrix}$$

Q6 Solution:

$$\lambda^3 + (-4)\lambda^2 + (5)\lambda - 2 = 0$$

Q7 Solution: $\mathbf{B}^5 = \begin{bmatrix} 1 & 10 & 145 \\ 0 & 1 & 31 \\ 0 & 0 & 32 \end{bmatrix}$

Q8 Hint: Check the eigenvector matrix & its inverse of the diagonalization formula.

Q9 Hint: Compare the efficiency in computing the power of matrix; Compare the efficiency in developing the formulation; Compare the complexity of data requires for executing the method; Compare the complexity of executing the method.

Q10:

Eigenvector or modal matrix, $\mathbf{P} = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{2} & 0 & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$

Eigenvalues or spectral matrix, $\mathbf{D} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Hint: For verification,

$$\mathbf{CP} = \mathbf{PD}$$