

KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 9: Application of integral

1. Find the area bounded by the curves.
 - (i) $y = x^4 - x^2$ and $y = x^2$ (the part to the right of the y-axis)
 - (ii) $x = y^3$ and $x = y^2$
 - (iii) $y = \cos(\pi x/2)$ and $y = 1 - x^2$ (in the first quadrant)
2. Find the area bounded by the parabola $x = 8 + 2y - y^2$, the y-axis, and the lines $y = -1$ and $y = 3$.
3. Assume a cylindrical tank of radius 4 m and height 10 m is filled to a depth of 8 m as shown Figure 3.1. How much work is needed to empty a tank partially filled with water?

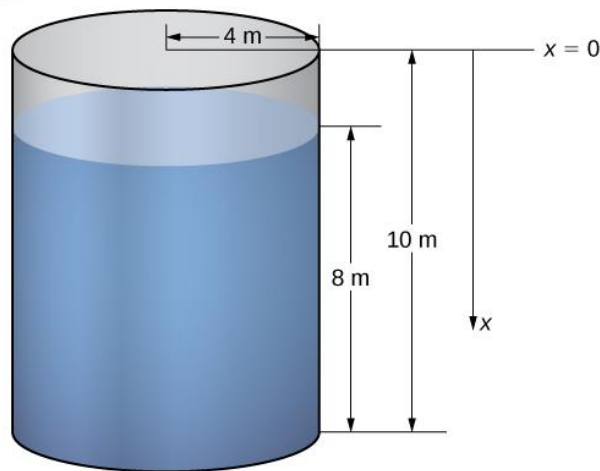


Figure 3.1 A tank partially filled with water

4. An object moves along a straight line with acceleration given by $a(t) = -\cos(t)$, and $s(0) = 1$ and $v(0) = 0$. Find the maximum distance the object travels from zero, and find its maximum speed. Describe the motion of the object.
5. A particle at rest leaves the origin with its velocity increasing with time according to $v(t) = 3.2t$ m/s. At 5.0 s, the particle's velocity starts decreasing according to $[16.0 - 1.5(t - 5.0)]$ m/s. This decrease continues until $t = 11.0$ s,

after which the particle's velocity remains constant at 7.0 m/s. (a) What is the acceleration of the particle as a function of time? (b) What is the position of the particle at $t = 2.0$ s, $t = 7.0$ s, and $t = 12.0$ s?

6. A force of 50 N will stretch a spring from its natural length of 2 meters to a length of 2.5 meters.

(i) Find the spring constant.

(ii) Find the work needed to stretch the spring by a length of 0.5 meter from its natural length.

7. Find the arc length L of the curve $y = x^{3/2}$ from $x = 0$ to $x = 5$.

8. Find the arc length of the catenary $y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$ from $x = 0$ to $x = a$.

9. Find the centroid of the plane area bounded by the parabolas $y = 2x - x^2$ and $y = 3x^2 - 6x$.

10. Find the hydrostatic force on a circular plate of radius 2 that is submerged 6 meters in the water.